M.Tech. [Online]: ECE - Written Test
Sample Test

Instructions:
All questions carry 1 mark each. There are no negative marks.

I. Engineering Mathematics Questions [Answer Any 8 Questions]

1) Coin 1 and Coin 2 show heads with probability 0.25 and 0.75 when tossed, respectively. Suppose a coin is chosen uniformly at random and tossed, and it shows heads. The probability that Coin 1 was chosen is
Answer:

2) Let A and B be two independent events with probabilities $\Pr[A] = 0.6$ and $\Pr[B] = 0.5$. Then, the probability $\Pr[A \cup B]$ equals
   a) 1
   b) 1.1
   c) 0.8
   d) 0.7

3) A coin shows heads with probability p when tossed. If the coin is tossed n times, then the probability of getting exactly one tail or exactly one head is
Answer:

4) $F_X(\cdot)$ and $F_Y(\cdot)$ denote the cumulative distribution function (CDF) of the random variables X and Y, respectively. X and Y are mutually independent. The probability of the event $\max\{X, Y\} \leq a$ is given by
   a) $\max\{F_X(a), F_Y(a)\}$
   b) $F_X(a) + F_Y(a)$
   c) $F_X(a)F_Y(a)$
   d) None of the above

5) If $\mathbf{A}$ is an $n \times n$ unitary matrix, what are the rank and determinant of $\mathbf{A}$?
   a) Rank $= 1$, determinant $= 0$
   b) Rank $= n$, determinant $= 1$
   c) Rank $= n$, determinant $= \pm 1$
   d) Cannot be determined from the information provided

6) Suppose $\mathbf{A}$ is an $n \times n$ matrix whose eigenvalues are $\{3, -1, 0, \ldots, 0\}$, where there are $n - 2$ zeros. What are the eigenvalues of $\mathbf{A}^2 + \mathbf{A}$?
   a) $9, 1$ and $n - 2$ zeros
   b) $8, 4$ and $n - 2$ zeros
   c) 12 and $n - 1$ zeros
   d) Cannot be determined from the information provided

7) What is the characteristic polynomial of the matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$?
   a) $x^2 + 1$
   b) $x^2 - 1$

c) $x^2 + 2x + 1$
d) $x^2 - 2x + 1$

8) Find the rank and nullity of $\mathbf{A} = [1, 2]$.
a) Rank $= 0$, nullity $= 0$
b) Rank $= 0$, nullity $= 1$
c) Rank $= 1$, nullity $= 0$
d) Rank $= 1$, nullity $= 1$

9) Consider the matrix $\begin{pmatrix} 1+\rho & \rho \\ \rho & 1+\rho \end{pmatrix}$, where $0 \leq \rho \leq 1$. What are its eigenvalues?
a) ρ and $1 + \rho$
b) ρ and $1 + 2\rho$
c) 1 and $1 + \rho$
d) 1 and $1 + 2\rho$

10) For the equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 + 3x - 2y = 0$, obtain the order and the degree
a) Order: 2, Degree: 1
b) Order: 2, Degree: 3
c) Order: 3, Degree: 2
d) Order: 1, Degree: 2

11) Identify the solution for the differential equation $\frac{d^2y}{dx^2} - 10\frac{dy}{dx} + 25y = 0$
a) $y = Ae^{5x}$
b) $y = Ae^{5x} + Be^{-5x}$
c) $y = A\ x\ e^{5x} + Be^{5x}$
d) $y = Ae^{5x} - Be^{5x}$

12) What is the solution of differential equation $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 30y = 0$?
Answer:

II. Subject Questions [Answer Any 12 Questions]

1) The transmitted signal X takes values -1 and 1 with equal probability. The received signal Y is equal to

$$Y = X + N, \tag{1}$$

where the noise N is independent of X and is uniformly distributed over $[-2, 2]$. For the maximum likelihood decoder, the probability of error (up to 1 decimal place) is equal to ?
Answer:

2) For a transmission scheme that uses 64-QAM and a coding rate of $5/6$, the number of information bits transmitted in a codeword of length 1000 symbols is
Answer:

3) Consider the constellation shown in Fig. 1. All points in the constellation are transmitted with equal probability. The average transmit energy is equal to
a) a^2
b) $3a^2/4$
c) a^4

d) $a^2/2$

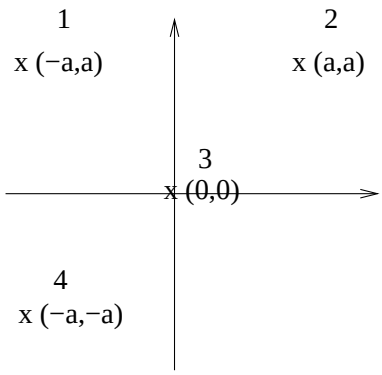

Fig. 1

4) An image uses 16×16 picture elements. Each element can take any of 8 possible intensity levels. The maximum entropy of this image will be
   a) 2048 bits
   b) 1024 bits
   c) 4096 bits
   d) None of the above

5) A source generates three symbols with probabilities 0.25, 0.25, and 0.50. It transmits them at a rate of 3000 symbols/second. Assuming independent generation of symbols, the average rate of the most efficient source encoder would be
   a) 6000 bits
   b) 4500 bits
   c) 3000 bits
   d) None of the above

6) Let X denote the outcome of a fair die, i.e., it can take one of the six values $\{1, 2, 3, 4, 5, 6\}$ with equal probability. What is the entropy of the random variable $Y = X^2 + 2X + 1$?
   a) $\log(6)$
   b) $\log(6)/6$
   c) $\log^2(6) + 2\log(6) + 1$
   d) $(\log^2(6) + 2\log(6) + 1)/6$

7) A continuous-time signal lies in the frequency band $|\omega| < 5\pi$. This signal is contaminated by a large sinusoidal signal of frequency 120π. The contaminated signal is sampled at a sampling rate of $\omega_s = 13\pi$. After sampling, at what frequency does the sinusoidal interfering signal appear?
   a) 2π
   b) π
   c) 3π
   d) None of the above

8) $X = \sum_{n=-\infty}^{\infty} \frac{\sin^2(Wn)}{\pi^2 n^2}$ evaluates to
   a) π
   b) W/π
   c) $\sin(W\pi)$
   d) 0

9) A discrete-time system has impulse response $h[n] = a^n u[n+2]$, with $0 < |a| < 1$ and $u[n]$ is a unit step function. Is this system
   a) BIBO stable, causal, and memoryless
   b) BIBO stable, not causal, and not memoryless
   c) Not BIBO stable, causal, and memoryless
   d) Not BIBO stable, not causal, and not memoryless

10) The Laplace transform of $s(t) = \sum_{n=0}^{\infty} u(t-n)$ with $u(t)$ being a unit step function is
   a) $\frac{1}{s(1 - e^{-s})}$
   b) $\frac{1}{s(1 + e^{-s})}$
   c) $\frac{s}{(1 + e^{-s})}$
   d) e^{-2s}

11) Consider a discrete-time sequence, $x[n]$, with a z-transform denoted by $X(z)$ with the region-of-convergence $R1 < |z| < R2$. What is the ROC for the z-transform of $x(-n)$?
   a) $R1 < |z| < R2$
   b) $R2 < |z| < R1$
   c) $1/R1 < |z| < 1/R2$
   d) $1/R2 < |z| < 1/R1$

12) Let $w(n)$ be a white Gaussian random process with variance N_0 and be the input of a stable LTI system, with transfer function $H(f)$. The power spectral density of the output of the LTI system equals
   a) N_0
   b) $N_0|H(f)|$
   c) $N_0|H(f)|^2$
   d) $N_0/|H(f)|^2$

13) A silicon sample is doped with 10^{17} Arsenic atoms/cm^3. The equilibrium hole concentration p_0 at 300K is approximately given by:
   a) 2.25×10^3 /cm^3
   b) 2.25×10^9 /cm^3
   c) 4.5×10^3 /cm^3
   d) 4.5×10^9 /cm^3

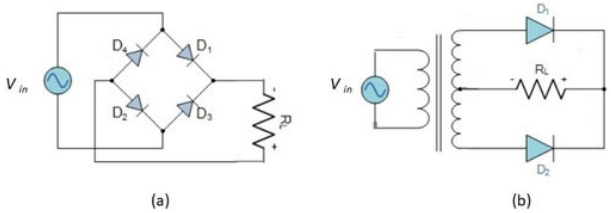

Fig. 2

14) If a sinusoidal input voltage V_{in} with frequency f_0 is applied to the two circuits shown in Fig. 2, the output voltage across the load resistor R_L has a large DC component along with a ripple. The frequency of the ripple is:
   a) f_0 for both circuits
   b) f_0 for circuit (a) and $2f_0$ for circuit (b)
   c) $2f_0$ for circuit (a) and f_0 for circuit (b)
   d) $2f_0$ for both circuits

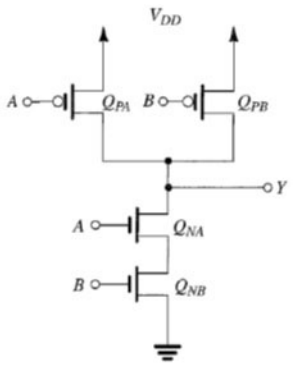

Fig. 3

15) For the CMOS logic circuit shown in Fig. 3, if $A = 1$ and $B = 1$, then the value of Y is:
    a) 0
    b) 1
    c) Unknown
    d) None of the above

16) The circuit diagram in Fig. 4 shows a CMOS D flip-flop. The clock phase provided to:
    a) TG1 is incorrect
    b) TG2 is incorrect
    c) TG3 is incorrect
    d) TG4 is incorrect

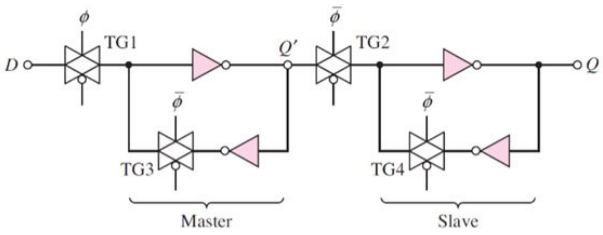

Fig. 4

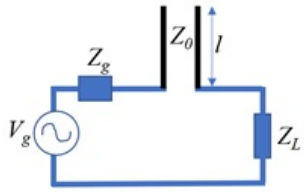

Fig. 5

17) Refer to the figure of the circuit in Fig. 5, where a 1 GHz generator with $Z_g = 50\ \Omega$ is connected to a load $Z_L = (50 - j30)\ \Omega$. In order to facilitate maximum power transfer to Z_L, the minimum length l for the open-circuited transmission line having characteristic impedance $Z_0 = 60\ \Omega$ would be approximately:
    a) 0.574λ
    b) 0.007λ
    c) 0.324λ
    d) 0.176λ

18) A certain two-port network has the following scattering matrix:

$$\mathbf{S} = \begin{bmatrix} 0.1\angle 0^o & 0.8\angle 90^o \\ 0.8\angle 90^o & 0.2\angle 0^o \end{bmatrix}$$

The network is:

a) Reciprocal and Lossy
b) Reciprocal and Lossless
c) Non-reciprocal and Lossless
d) Non-reciprocal and Lossy